Roll No ....

MEEM-101

M.E./M.Tech., I Semester Examination, June 2020

Applied Mathematics

Time : Three Hours

Maximum Marks : 70

Note: i) Attempt any five questions.

ii) All questions carry equal marks.

1. a) Solve $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 1 - e^{2t}$, $y = 1$, $\frac{dy}{dt} = 0$ at $t = 0$, by using Laplace transform.

   b) Solve the integral equation with the help of Laplace transform.

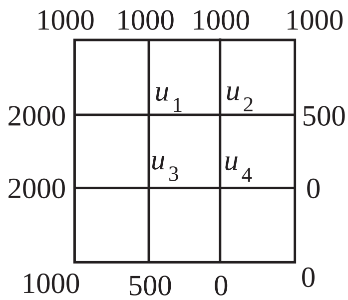
   $$F(t) = e^{-t} - 2\int_0^t \cos(t-u) \cdot F(u)\, du$$

2. a) Find Fourier sine transform of $\frac{e^{-ax}}{x}$.

   b) Using modified Euler's method, find an approximate value of y when $x = 0.3$, given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$.

3. a) Find the values of $u(x, t)$ satisfying the parabolic equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ and the boundary conditions $u(0, t) = 0 = u(8, t)$ and $u(x, 0) = 4x - \frac{1}{2}x^2$ at the points $x = i : i = 0, 1, 2, \ldots, 7$ and $t = \frac{1}{8} j : j = 0, 1, 2, \ldots, 5$.

   b) Given the values of $u(x, t)$ on the boundary of the square in the following figure, evaluate the function $u(x, t)$ satisfying the Laplace equation $\nabla^2 u = 0$ at the pivotal points of this figure by Gauss-Seidal method.



4. a) In a precision bombing attack there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target?

   b) In 1,000 extensive sets of trials for an event of small probability the frequencies f of the number x of successes are found to be

x :	0	1	2	3	4	5	6	7
f :	305	365	210	80	28	9	2	1

   Assuming it to be a Poisson distribution, calculate its mean, variance and expected frequencies for Poisson distribution with same mean.

5. a) Find the mean and variance for Binomial distribution.

   b) In two large populations there are 30% and 25% respectively of fair haired people. Is this difference likely to be hidden in sample of 1200 and 900 respectively from the two populations?

6. a) Using Runga Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, with $y(0) = 1$ at $x = 0.2, 0.4$

   b) Define standard and probable error and what are the different standard errors?

7. a) Show that $Z\left(1/\underline{|n}\right) = e^{1/z}$. Hence evaluate $Z\left[1/\underline{|n+1}\right]$ and $Z\left[1/\underline{|n+2}\right]$.

   b) Find the inverse Z-transform of $\frac{z^3 - 20z}{(z-2)^3(z-4)}$.

8. a) Suppose the population of the world now is 4 billion and it is doubling period is 35 years, what will be the population of the world after 350 years, 700 years, 1050 years? If the surface area of the earth is 1,860,000 billion square feet, how much space would each person get after 1050 years?

   b) Discuss the following:

      i) Logistic law of population growth

      ii) Population growth model.

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